

GCSE Maths – Ratio, Proportion and Rates of Change

Interpreting Gradients (Higher Only)

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of interpreting gradients questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

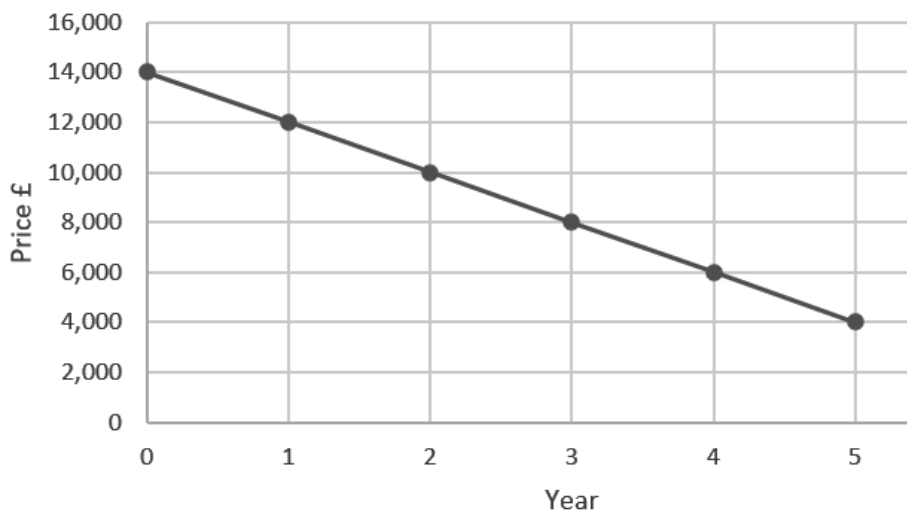
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Section A

Worked Example

Depreciation of car value



Calculate the rate of change between the price of the car and the year.

Step 1: Find two co-ordinates' points on this graph.

$$(x_1, y_1) = (0, 14000)$$

$$(x_2, y_2) = (5, 4000)$$

Step 2: Calculate the gradient using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$.

$$m = \frac{14000 - 4000}{0 - 5}$$

$$m = \frac{10,000}{-5} = -2000$$

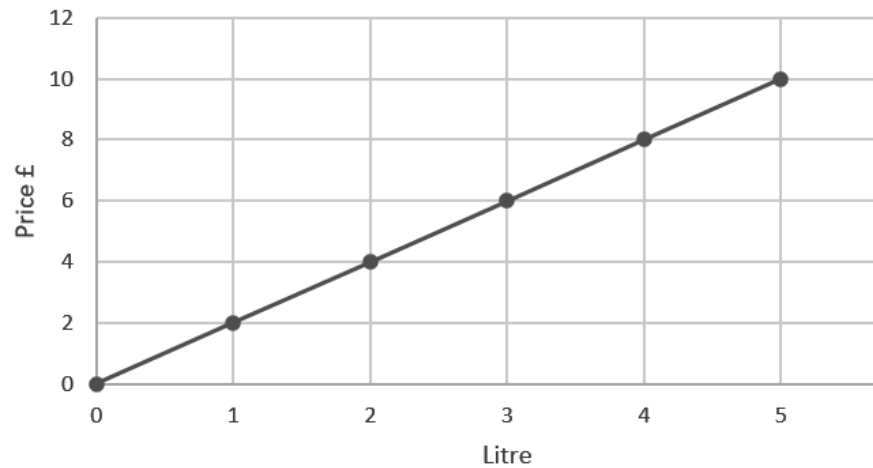
Step 3: Form a conclusion.

The rate of change between the car value and year is -2000 . The car depreciates **£2000** every year.



Guided Example

Price of oil per litre



Calculate the rate of change between the price of oil and litre (cost per litre).

Step 1: Find two co-ordinates' points on this graph.

$$(x_1, y_1) = (5, 10)$$

$$(x_2, y_2) = (0, 0)$$

Step 2: Calculate the gradient using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$.

$$m = \frac{10 - 0}{5 - 0} = \frac{10}{5} = 2$$

Step 3: Form a conclusion.

The rate of change between price of oil and litre is 2. The cost per litre of oil is £2.

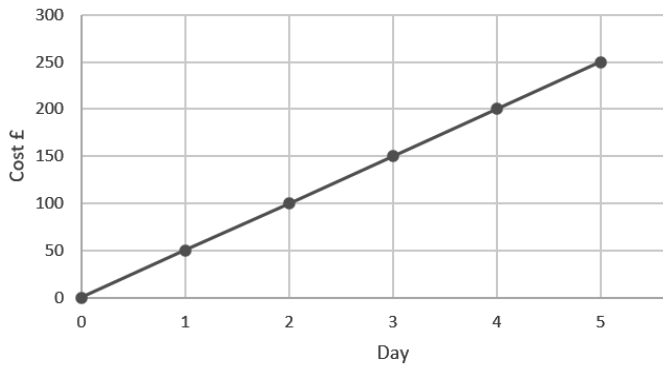


Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Calculate the cost per day for renting a hotel room.

Cost of renting a hotel room



$$(x_1, y_1) = (5, 250)$$

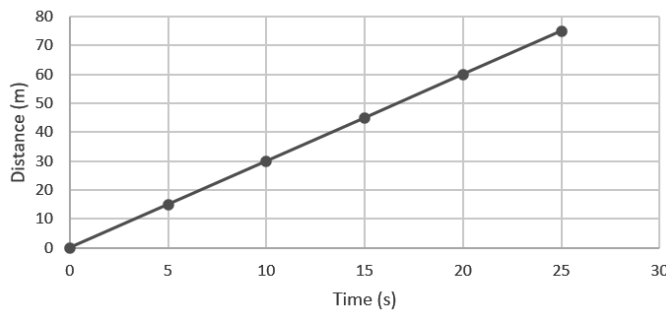
$$(x_2, y_2) = (0, 0)$$

$$m = \frac{250 - 0}{5 - 0} = \frac{250}{5} = \frac{50}{1} = 50$$

The cost per day is £50.

2. Calculate the rate of change between the distance and time (speed).

The relationship between the distance and time of a toy car



$$(x_1, y_1) = (20, 60)$$

$$(x_2, y_2) = (10, 30)$$

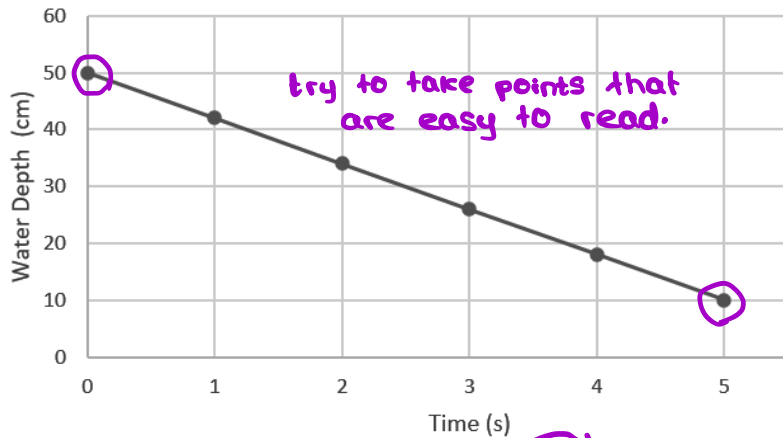
$$m = \frac{60 - 30}{20 - 10} = \frac{30}{10} = \frac{3}{1} = 3$$

The speed of the toy car is 3 m/s



3. Calculate the rate of water discharge.

Rate of water flowing out of container



$$(x_1, y_1) = (5, 10)$$

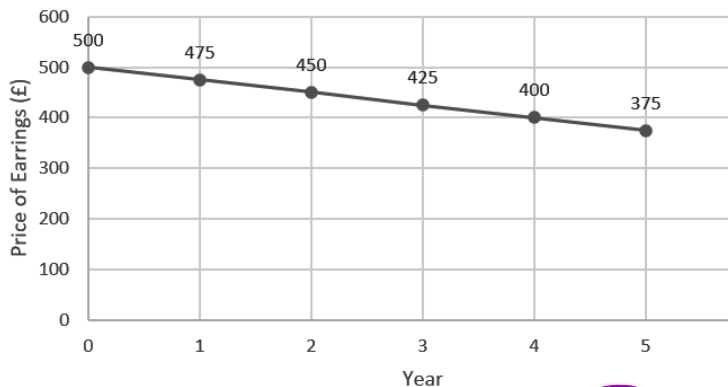
$$(x_2, y_2) = (0, 50)$$

$$m = \frac{10 - 50}{5 - 0} = \frac{-40}{5} = \frac{-8}{1} = -8$$

The rate of water discharge is -8 cm/s

4. Calculate the rate of depreciation of the antique earrings.

Depreciation of earrings



$$(x_1, y_1) = (0, 500)$$

$$(x_2, y_2) = (4, 400)$$

$$m = \frac{500 - 400}{0 - 4} = \frac{100}{-4} = -\frac{25}{1} = -25$$

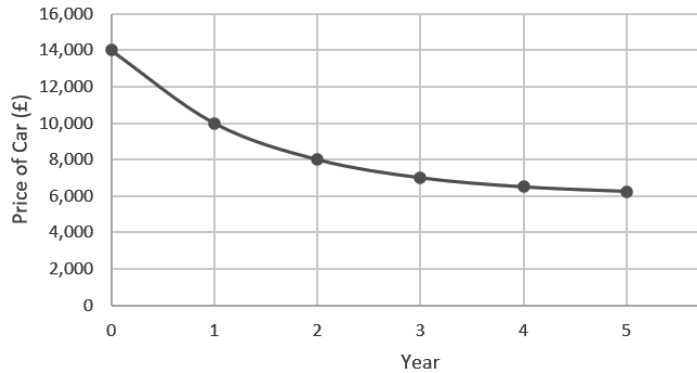
The rate of depreciation of antique earrings is £25 per year.



Section B

Worked Example

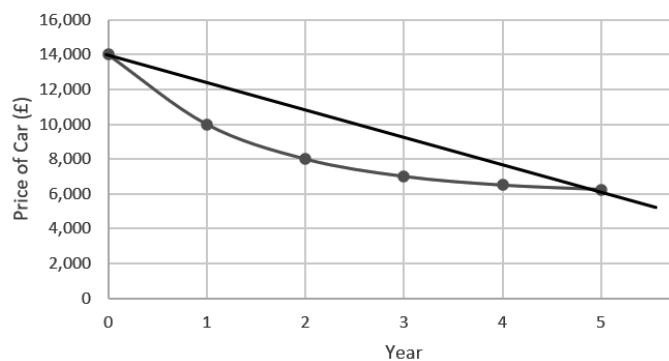
Depreciation of car value



Estimate the average rate of change between the car value and year.

Step 1: Draw a chord using a ruler and pencil, connecting the starting and ending point.

Depreciation of car value



Step 2: Find two co-ordinates' points on this chord:

$$(x_1, y_1) = (0, 14000)$$

$$(x_2, y_2) = (5, 6250)$$

Step 3: Calculate the gradient using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$.

$$m = \frac{14000 - 6250}{0 - 5} = \frac{7750}{-5} = -1550$$

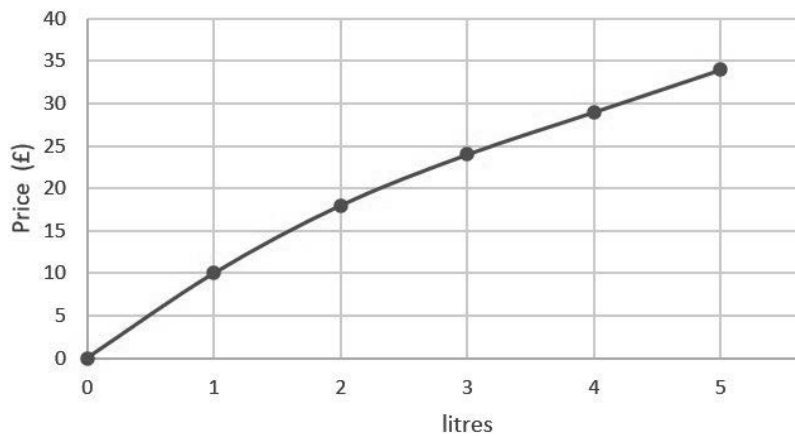
Step 4: Form a conclusion.

The estimate of the rate of change between the car value and year is -1550 . The car depreciates £1550 every year approximately.



Guided Example

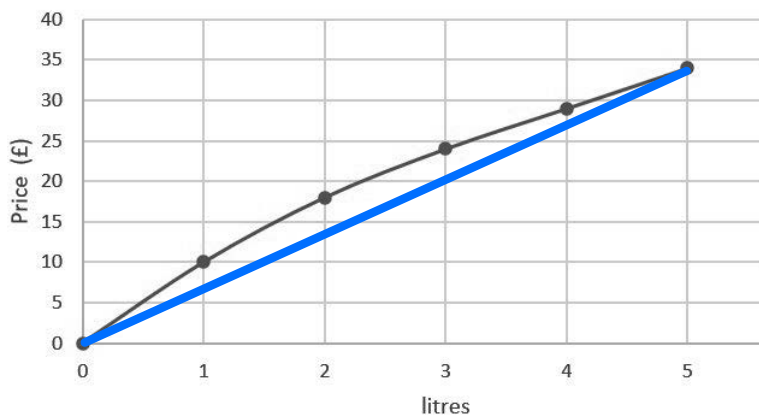
Cost of oil per litre



Estimate the cost of oil per litre.

Step 1: Draw a chord using a ruler and pencil, connecting the starting and ending point.

Cost of oil per litre



Step 2: Find two co-ordinates' points on this chord.

$$(x_1, y_1) = (5, 34)$$

$$(x_2, y_2) = (0, 0)$$

Step 3: Calculate the gradient using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$.

$$m = \frac{34 - 0}{5 - 0} = \frac{34}{5} = \frac{68}{10} = 6.8$$

Step 4: Form a conclusion.

The estimated cost of oil per litre is £6.8

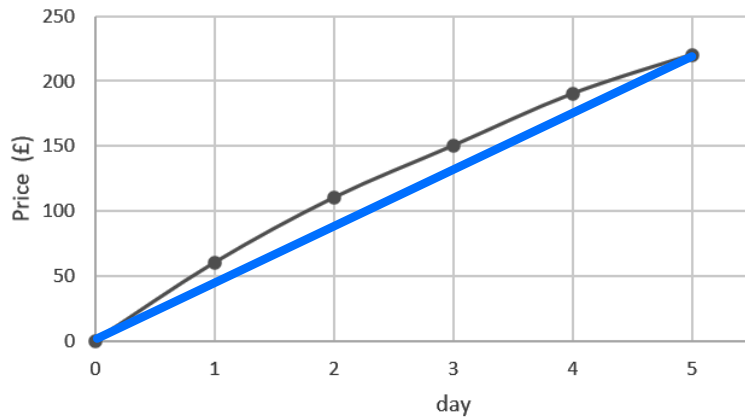


Now it's your turn!

If you get stuck, look back at the worked and guided examples.

5. Estimate the cost per day for renting a hotel room.

Cost of renting a room per day



$$(x_1, y_1) = (5, 220)$$

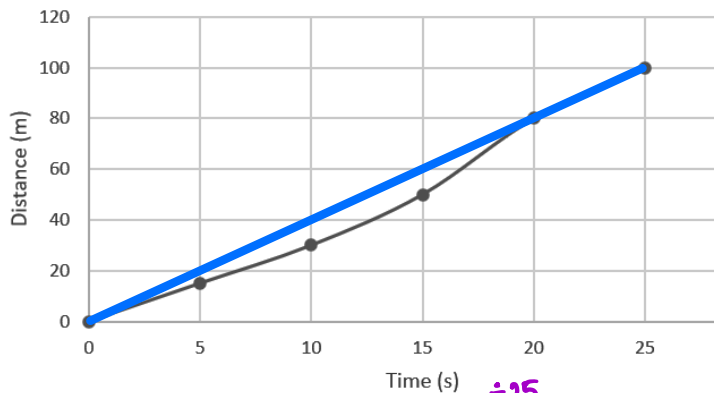
$$(x_2, y_2) = (0, 0)$$

$$m = \frac{220 - 0}{5 - 0} = \frac{220}{5} = \frac{440}{10} = 44$$

The estimated cost of renting a room per day is £44.

6. Estimate the rate of change between the distance and time (speed)

Distance and time travelled by a toy car



$$(x_1, y_1) = (25, 100)$$

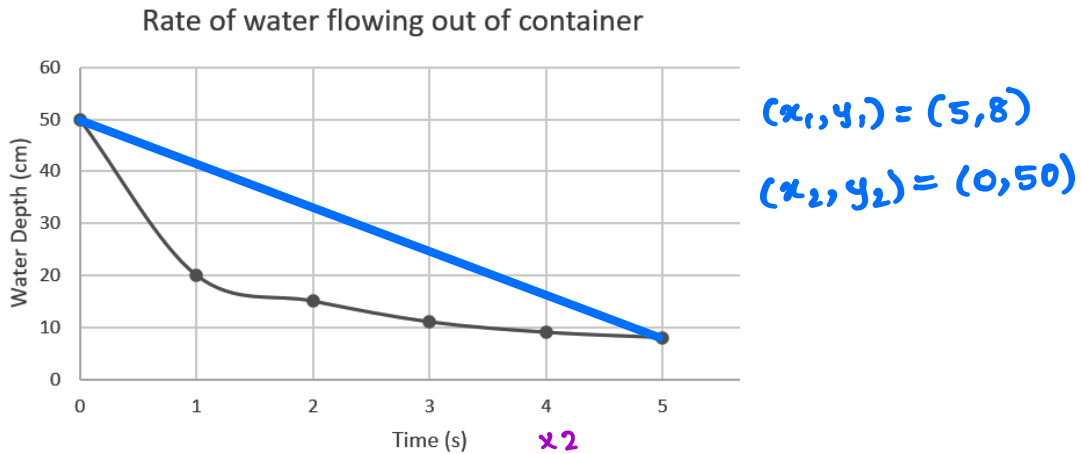
$$(x_2, y_2) = (0, 0)$$

$$m = \frac{100 - 0}{25 - 0} = \frac{100}{25} = \frac{4}{1} = 4$$

Estimated Speed of toy car is 4 m/s



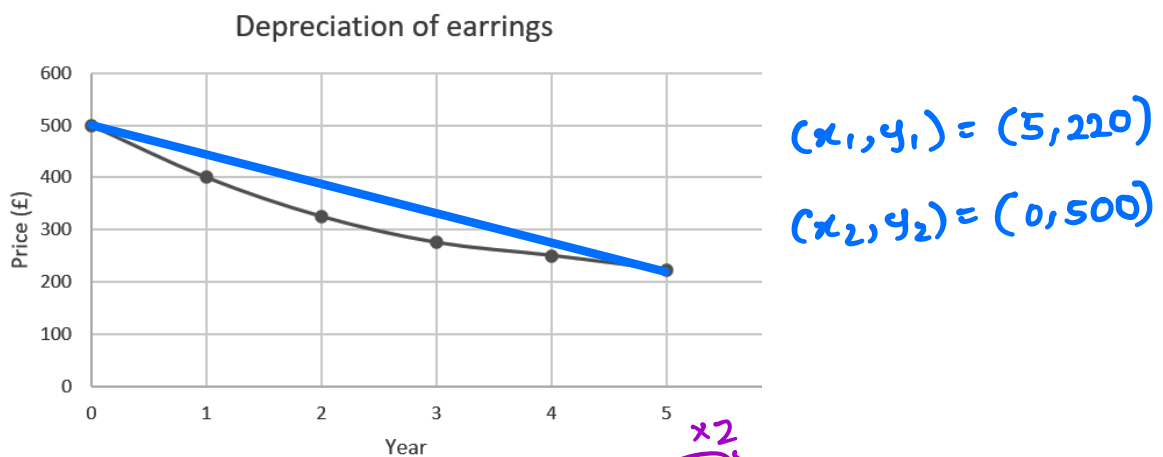
7. Estimate the rate of water discharge.



$$m = \frac{8 - 50}{5 - 0} = \frac{-42}{5} \stackrel{\times 2}{=} \frac{-84}{10} = -8.4$$

Estimated rate of water discharge is 8.4 cm/s

8. Estimate the depreciation of the antique earrings



$$m = \frac{220 - 500}{5 - 0} = \frac{-280}{5} \stackrel{\times 2}{=} \frac{-560}{10} = -56$$

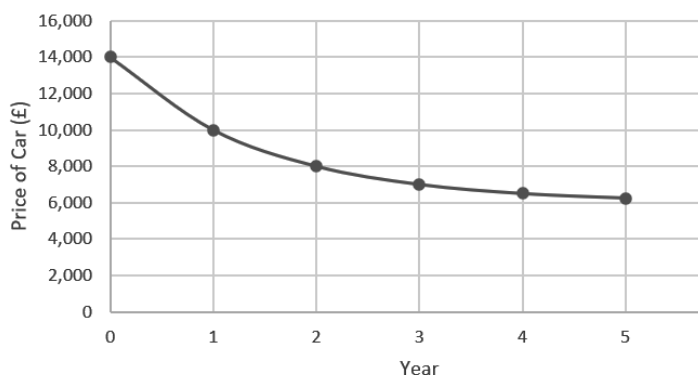
The estimated annual depreciation of the antique earrings is £56



Section C

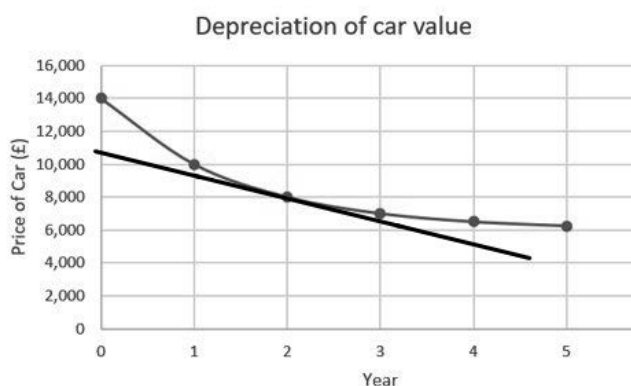
Worked Example

Depreciation of car value



Estimate the rate of change between the car value and year in year 2.

Step 1: Draw a tangent using a ruler and pencil at year 2.



Step 2: Find two co-ordinates' points on this tangent:

$$(x_1, y_1) = (1, 9500)$$

$$(x_2, y_2) = (2, 8000)$$

Step 3: Calculate the gradient using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$.

$$m = \frac{9500 - 8000}{1 - 2} = \frac{1500}{-1} = -1500$$

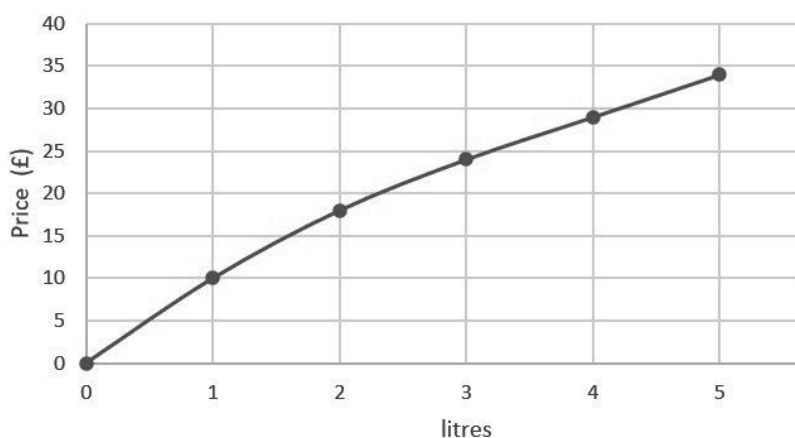
Step 4: Form a conclusion.

At year 2, the depreciation of the car is approximately **£1500** per year.



Guided Example

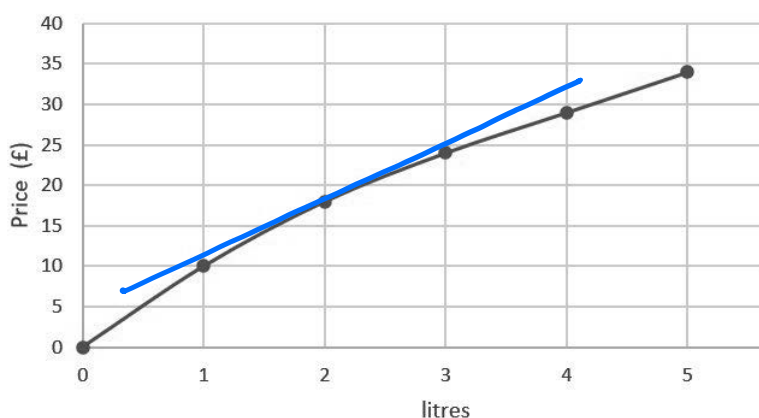
Cost of oil per litre



Estimate the cost of oil per litre when there are 2 litres of oil.

Step 1: Draw a tangent using a ruler and pencil at litre 2.

Cost of oil per litre



Step 2: Find two co-ordinates' points on this tangent.

$$(x_1, y_1) = (2, 18)$$

$$(x_2, y_2) = (3, 25)$$

Step 3: Calculate the gradient using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$.

$$m = \frac{18 - 25}{2 - 3} = \frac{-7}{-1} = 7$$

At two litres of oil, the price of a litre is approximately £7.

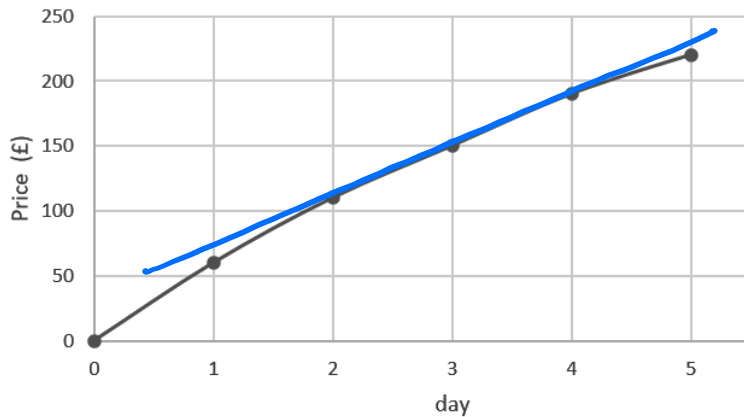


Now it's your turn!

If you get stuck, look back at the worked and guided examples.

9. Estimate the cost per day for renting a hotel room on day 3.

Cost of renting a room per day



$$(x_1, y_1) = (4, 190)$$

$$(x_2, y_2) = (1, 75)$$

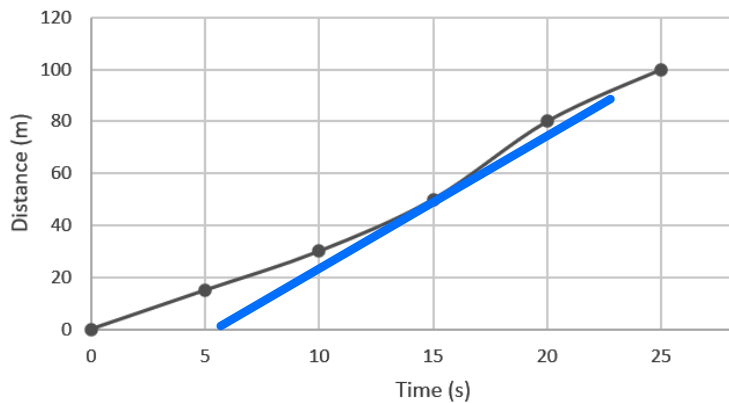
$$m = \frac{190 - 75}{4 - 1} = \frac{115}{3} = 38.3$$

$$3 \overline{)115} \begin{array}{r} 38.3 \\ 9 \\ \hline 25 \\ 24 \\ \hline 10 \\ 9 \\ \hline 10 \end{array}$$

The estimated cost of renting a room on day 3 is £38.3

10. Estimate the speed at 15 seconds.

Distance and time travelled by a toy car



$$(x_1, y_1) = (20, 75)$$

$$(x_2, y_2) = (6, 0)$$

$$m = \frac{75 - 0}{20 - 6} = \frac{75}{14} = 5.36$$

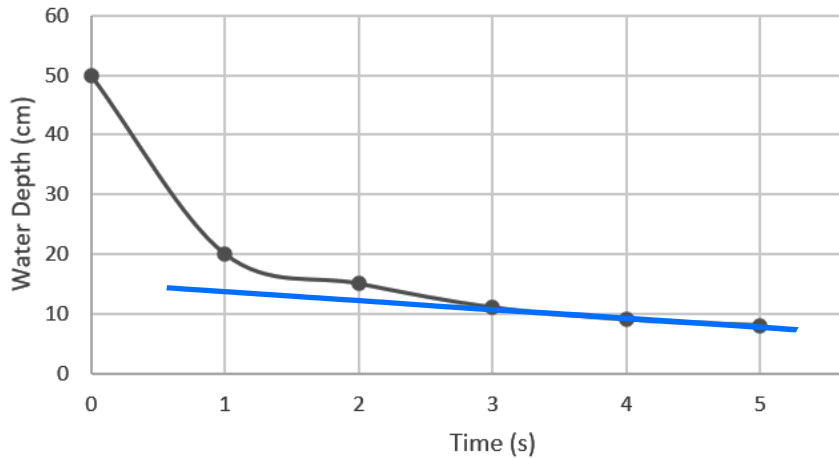
$$14 \overline{)75} \begin{array}{r} 5.357 \\ 70 \\ \hline 50 \\ 42 \\ \hline 80 \\ 70 \\ \hline 100 \\ 98 \\ \hline 20 \end{array}$$

Estimated Speed of toy car at 15 seconds is 5.36 m/s



11. Estimate the rate of water discharge at 4 seconds.

Rate of water flowing out of container



$$(x_1, y_1) = (5, 8)$$

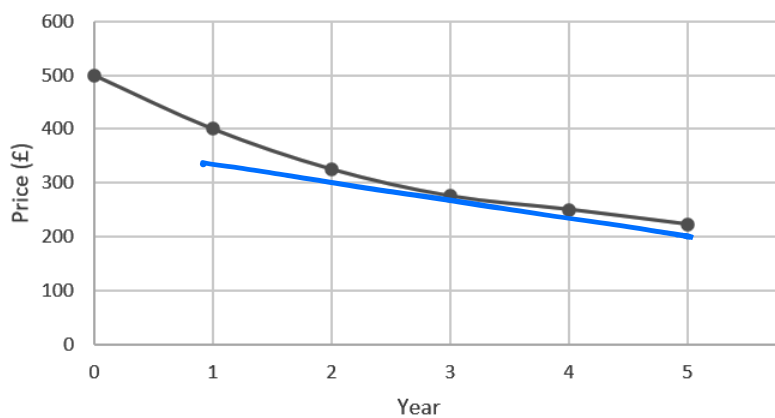
$$(x_2, y_2) = (1, 14)$$

$$m = \frac{8 - 14}{5 - 1} = \frac{-6}{4} = \frac{-3}{2} = -1.5$$

Estimated rate of water discharge at 4 seconds is 1.5 cm/s

12. Estimate the rate of depreciation of the antique earrings in Year 3.

Depreciation of earrings



$$(x_1, y_1) = (5, 200)$$

$$(x_2, y_2) = (2, 300)$$

$$m = \frac{200 - 300}{5 - 2} = \frac{-100}{3} = -33.3$$

$$3 \overline{) 100} \begin{array}{r} 33.3 \\ 9 \\ \hline 10 \\ 9 \\ \hline 10 \end{array}$$

Estimated annual depreciation of antique earrings Year 3 is £33.3

